2101. Write m as $2k$, for $k \in \mathbb{N}$, and then find an explicit factorisation. Consider how the $m = 2$ case arises as an exception to your argument.

2102. Find intersections, remembering that the domain of definition of the square root function is non-negative reals. Then set up a definite integral.

2103. (a) Differentiate twice and then use Pythagoras.
 (b) Rewrite the position as a quadratic in t, whose coefficients are vectors.

2104. The geometry is easiest if you set the side length to 1. Then use the standard results for polygons, and the cosine rule.

2105. For each set, solve the inequality and hence find the intersection over the reals $\mathbb{R}$. Then consider which integers lie in the interval.

2106. (a) i. Find $\mathbb{P}(M < 1)$ using a calculator normal distribution.
 ii. Use the probability from part (a).
 (b) What allows you to multiply probabilities?

2107. Find (p, q) by completing the square. Transform your parabola (still in completed square form) to make it positive, with its vertex unchanged.

2108. Any contact force can be considered as the vector sum of a reaction force which is perpendicular to the surface and a frictional force which is parallel to it.

2109. Consider the number of arrangements of $\text{A}_1\text{R}_1\text{R}_2\text{A}_2$NGED, in which the As and Rs are seen as different letters. Then work out how many of these arrangements spell the word ARRANGED.

2110. Use the double-angle formula linking $\cos 2x$ and $\sin^2 x$. Then you can consider this graph as a transformation of the $y = \cos x$ graph.

2111. In the first equation, make $\sec t$ the subject, and square both sides. Do likewise with the second equation. Then substitute both into the second Pythagorean trig identity.

2112. Don't use the formula for the nth term here. It's easier to make a single equation, translating the English "The ratio between the first and second terms is the same as the ratio between the second and third terms."

2113. (a) Use the product and chain rules to find the first derivative. Take out a factor of $(3x - 2)^2$ and simplify. Then differentiate again, and take out a factor of $(3x - 2)$. Explain how you know that the second derivative changes sign at $x = 2/3$.

 — Alternative Method —

 Use the fact that the factor $(3x - 2)$ is cubed.
 (b) Use the discriminant to show that the first derivative has only one root.
 (c) Use (a) and (b), together with the fact that there are no x intercepts other than the triple root at $x = 2/3$.

2114. Consider two scenarios, one in which T will be maximal, one in which T will be minimal. In each case, there is only one possible arrangement of the other three forces.

2115. Consider $a = 0$ and $a = -\pi/2$.

2116. A root is an intersection of $y = \text{f}(x)$ with the x axis, while a fixed point is an intersection of $y = \text{f}(x)$ with the line $y = x$. Find a curve which has the former but not the latter.

2117. (a) Substitute $x = 0$ and $x = 2$ into the curve to find the coordinates of the two endpoints. Then find the gradient m of the chords and use $y - y_1 = m(x - x_1)$.
 (b) Differentiate the equation of the curve. Set the derivative to m and solve with your calculator. You should find three roots, one of which is the required point of tangency.

2118. Use either the change of base formula, or the fact that $\log_{a^2} b^2 = \log_a b$, to express every logarithm with base 4. Then $\log_4 p = \log_4 q \implies p = q$.

2119. Since height is assumed to be continuous, there are fifty people above median height. The situation is binomial without replacement: the probability changes as individuals are chosen. Since there is only one successful outcome, you need to multiply five probabilities together.

2120. Take the LHS as an expression. Simplify using the binomial expansion. Then solve the equation. There are square roots involved, so be careful to check the validity of any roots you find.

2121. This is correct: explain why it is so with reference to Newton's third law.

2122. You need only prove the result for a_1. Symmetry means that the result for a_2 follows. Begin with "Assume, for a contradiction, that $\text{f}'(a_1) = \text{g}'(a_1)$." Then consider the roots (single/double) of the equation $\text{f}(x) = \text{g}(x)$.

2123. (a) The null hypothesis is $H_0 : \mu = 1.1802$.
   (b) The test is two-tail, so the critical region should consist of two regions, one at each tail.
   (c) The "test statistic" is the value of the sample mean. If this lies in the critical region, then there is sufficient evidence to reject the null hypothesis.

2124. The statement is true. Algebraically, show that the two equations for fixed points are the same.

——— Alternative Method ———

Thinking graphically, consider what the following have in common:

- fixed points,
- the transformation between $y = \text{f}(x)$ and $x = \text{f}(y)$.

2125. (a) Draw force diagrams, labelling acceleration on each. Form two equations of motion, and solve simultaneously for a. You'll need to rearrange to get the required result.
   (b) What physical assumption allows you to model the acceleration of the two masses as being equal?
   (c) Consider the limit of (a) as $k \to \infty$.

2126. The parabola must have its vertex on the x axis. This means that it must have a double root on the x axis. Consider the two factors $(ax + b)$ and $(cx + d)$ in light of this fact.

2127. Translate the English into algebraic statements, using the intersection definition of independence.

2128. In (a) and (b), you can simply take the limit, i.e. set x to the value towards which it is tending. However, you can't do this in (c), as $\frac{\infty}{\infty}$ is not well defined. Instead, divide top and bottom by 2^x before taking the limit.

2129. This isn't true. Consider the function $\text{f}(x) = \frac{-1}{x}$, defined everywhere except $x = 0$.

2130. (a) Write out the algebra and simplify.
   (b) Take each side of the non-identity separately. Simplify each separately, until they are clearly non-identical.

   ——— Alternative Method ———

   Provide a counterexample: values of a, b, c for which the two sides are not equal.

2131. (a) This is because a census is a sample which is, or attempts to be, the whole population.
   (b) Describe the strength of the correlation.
   (c) Will this pupil's reinstatement strengthen the correlation between the variables or weaken it?

2132. Simply substitute the expression $ax + b$ in for $\text{f}(x)$ and carry out the two integrals. Your answers, both in terms of a and b, should be identical.

2133. The possibility space contains $6^3 = 216$ outcomes. List/count the successful outcomes.

2134. This is true is some circumstances, depending on the object to which NII is being applied. Think about applying NII to

- the whole system of both objects,
- one of the objects.

2135. Begin with the LHS as an expression. Multiply it out and group the terms to reach the RHS.

2136. Find the second derivative f'', and show that f and f'' always have the same sign.

2137. This is false. There are counterexamples where the solution set is empty, and counterexamples where the solution set has infinitely many elements.

2138. (a) Quote the relevant sentence in the question that says "Friction is maximal."
   (b) Draw a force diagram, assuming the result in (a). For the moments equation, the relevant perpendicular distances are $2\sin\theta$ and $\cos\theta$.
   (c) Find R_{wall} in terms of mg. Then substitute into the moments equation. Divide both sides by $\cos\theta$ to get $\tan\theta$.

2139. Consider the arbitrary constants generated when finding an anti-derivative.

2140. Use the chain rule to differentiate: show that the result isn't true.

2141. Sketch the scenario first, drawing the radius to a point of tangency. Give the radius length 1. Then calculate the areas of both shapes and set up the ratio.

2142. Multiply up by the denominators. Then equate coefficients of x^0 and x^2.

2143. An acute triangle is one in which all three angles are acute. So, prove that the largest angle, which is opposite the largest side $n + 2$, is acute. This involves making $\cos\theta$ the subject of the cosine rule and simplifying.

2144. (a) Either proceed algebraically, by finding the first and second derivatives, or else consider the curve $y = \mathrm{f}(x) - 4$.
(b) Substitute the point $(2, 28)$ and solve for a.

2145. This is a quadratic in $\sin x$. Factorise it, and then determine the number of roots of each of the two equations so produced.

2146. The range is the set of outputs of the function. So, the scale factors at the front don't complicate matters very much; they merely scale the results down. The main issue is the parity of the power: odd powers retain the sign of the input, while even powers do not.

2147. This can be solved by resolving into components, but is most easily solved using a triangle of forces. Note that $(1.5, 2, 2.5)$ is a right-angled triangle.

2148. The set $\mathbb{Q}^+$ is the set of positive rationals. So, consider the input $x = 2$.

2149. Note that the factor of $\frac{1}{n^2}$ is constant as far as the index i is concerned. Hence, it can be taken out of the sum, leaving a standard arithmetic series result.

2150. As in many probability problems, consider firstly whether or not A and B can be guaranteed to be independent.

2151. Set up the first-principles differentiation using the standard formula:

$$\frac{d}{dx}\left(\frac{1}{x}\right) = \lim_{h\to 0}\frac{\frac{1}{x+h} - \frac{1}{x}}{h}.$$

Then, multiply top and bottom by $x(x+h)$. This allows simplification and cancellation of a factor of h. At this point you can safely take the limit.

2152. Show that the second derivative is positive for all values of x.

2153. Rewrite as $\sec^2 5x$. Then use a standard result and the reverse chain rule.

2154. The greatest vertical displacement occurs when the vertical velocity is zero. Use a vertical *suvat* to find the displacement y.

2155. Draw a sketch and use a vector method. The key fact is that a line joining midpoints is parallel to a line joining vertices.

2156. Factorise, and consider the repeated nature of one of the roots.

2157. (a) $(x - 3)$ could be positive or negative: this is why inequalities more complicated than linear are generally best solved using the boundary equation.
(b) Set the LHS equal to 1.
(c) Express the numerator as $x - 3 + 1$, then split the fraction up.

2158. (a) Since the situation is symmetrical in $y = x$, you can solve $x^2 - x = x$.
(b) This is easiest if you split it in two. The graph is symmetrical in $y = x$; find the area between $y = x^2 - x$ and $y = x$, then double it.

2159. Find an explicit iteration which is periodic, but has no fixed points. You might want to consider $u_{n+1} = \mathrm{f}(u_n)$, where f is a function whose domain is discrete, i.e. a list of elements.

2160. "Verify" means check using the given answer, as opposed to "show", which means derive from the ground up. So, find the derivative of the proposed solution, substitute into the LHS (as an expression) and simplify to get $3x$.

2161. (a) Compare the integral from -1 to 0 with the integral from 0 to -1.
(b) Split the integral up before evaluating the two parts.

2162. A negative cubic is the simplest example.

2163. Rearrange to $\lambda_1 = 1 - \lambda_2$. Then write the position vector $\overrightarrow{OP}$ as $\overrightarrow{OA} + \overrightarrow{AP}$.

2164. Two are false and one is true.

2165. Assume, for a contradiction, that $a, b, c \in \mathbb{N}$ are a Pythagorean triple, where a, b, c have no common factors and c is even.

Show that a and b must either both be odd or both be even. Consider the two cases separately, finding a different contradiction in each case.

2166. This is implicit differentiation, which is simply an example of the chain rule.

2167. Find a counterexample. Any cubic with strictly less than two stationary points will do.

2168. The question says "determine" rather than "find", so you need to do this algebraically. Set up an equation and solve, noting that, if you divide by p or $(1 - p)$, you should justify the step.

2169. Differentiate implicitly, then multiply out the brackets.

2170. Consider the line of symmetry of the parabola, i.e. the x coordinate of the vertex.

2171. Consider first whether the identities $|x|^2 \equiv x^2$ and/or $|x|^3 \equiv x^3$ are true. Then assess whether the sin, cos and tan graphs have $x = 0$ as a line of symmetry, like $y = x^2$, or not, like $y = x^3$.

2172. Find a function g which is decreasing everywhere except for at input zero, where it is stationary.

2173. Take the factor out manually by considering the coefficient of the first and last terms. You don't need to find the value of k.

2174. Consider the number of cards in the 1st, 2nd, 3rd, ... storey as an arithmetic progression. Calculate the sum with the standard formula for the sum of an AP.

2175. Solve to find A and k using the first two data points, and show that the curve you find doesn't go through the third point.

— Alternative Method —

In an exponential model, if the input variable x increases in AP, then the output variable y should increase in GP.

2176. This can be done by substitution or by parts. If you haven't yet learned either of these techniques, you could instead *verify* the answer to this problem by differentiation:

(a) Verify that

$$\frac{d}{dx}\left((6x-1)(4x+1)^{\frac{3}{2}}\right) = 60x\sqrt{1+4x}.$$

(b) Hence, show that

$$\int_0^1 60x\sqrt{1+4x}\,dx = 1 + 25\sqrt{5}.$$

— Alternative Method —

For integration by substitution, let $u = 1 + 4x$. This gives $dx = \frac{1}{4}du$ and also $60x = 15(u-1)$. Substitute for all instances of x, remembering to change the limits to $u = 1$ and $u = 5$. Multiply out and integrate definitely with respect to u.

— Alternative Method —

By parts, let $u = 60x$ and $v' = \sqrt{1+4x}$. Then $u' = 60$. Find v by the reverse chain rule. Then quote the integration by parts formula.

2177. For vertical asymptotes, the denominator must be zero. Note that $2k + 1$ is an odd number.

2178. (a) Condition the tree diagram on whether the leaves are infected.
(b) Sum the probabilities of two branches.
(c) Restrict the possibility space to the branches that fall early.

2179. Firstly, set $a = b = 0$, and sketch the region. Then, when you reinstate a and b, you can consider the new graphs as translations of the old graphs.

2180. Use calculus to find the equation of the tangent. Then solve this equation simultaneously with that of the curve, noting that the point $x = -1$ must be a root of the equation so formed. You might want to use the substitution $z = x^{\frac{1}{3}}$.

2181. Factorise the top and the bottom.

2182. In each case simplify using index laws.

2183. Draw a table of the 36 outcomes, and consider how restricting the possibility space to those outcomes with A prime changes the proportion of outcomes in which $|A - B| = 1$.

2184. Consider a specific case first, e.g. $x^3+4x^2+4x = 0$. In two of the cases, the answer is yes.

2185. Sketch the region carefully, with two boundary curves. Consider the y distance between the two curves. You might want to refer to an integral.

2186. A number ends 00 if and only if it has $10^2 = 2^2 \times 5^2$ as a factor.

2187. (a) You can write the answer down.
(b) This is projectile motion: use a vertical *suvat* to calculate the time of flight, then substitute back into a horizontal $d = vt$.
(c) Same again, only this time you need to resolve the initial velocity into horizontal and vertical components.

2188. It doesn't affect r. Consider the coding $y_i \mapsto ay_i+b$ as a graphical transformation of (x_i, y_i) points in the plane. Remember that the coefficient r is a measure of closeness to a *linear* relationship.

2189. Much of this is correct, but $y = \arcsin x$ is not *all* of $x = \sin y$.

2190. Draw a careful sketch. The centre of the triangle must lie at the centre of the square.

2191. Find the point of inflection by setting the second derivative to zero. Then substitute the x value back into the function.

2192. (a) Just differentiate $u = x^2$.
   (b) Use the chain rule.
   (c) "With respect to Q" can be translated as "as Q changes" or "relative to Q" or "from the point of view of Q".

2193. (a) i. Choose the first card wlog.
   ii. No calculation is needed.
   (b) Here, the question is whether the conditional probability in ii. is the same as the absolute probability in i.

2194. Assume, for a contradiction, that such a rectangle exists. Consider one of its pairs of parallel sides. Sketch these on $y = x^2$, and look at the distances between the points.

2195. These are inverse functions, so at least one of the implication arrows holds. The question is whether the domain of $y = e^x$ needs to be restricted in order for it to be invertible.

2196. Factorise to $y = x^3(x + 1)^3$. Then consider the location of the x intercepts.

2197. Use a double-angle formula to write $\cos 4x$ in terms of $\sin^2 2x$. Then solve.

2198. (a) Calculate the probability using the binomial distribution and its normal approximation $Y \sim N(np, npq)$. Since the binomial is the known distribution, use the binomial as the baseline for percentage error.
   (b) The normal approximation to the binomial only holds if neither np nor nq is small.

2199. Translate the given equation into algebra. Carry out the integrals, and solve for k.

2200. Divide numerator and denominator by x, before taking the limit.

——— End of 22nd Hundred ———